SUBJECT: DISCRETE MATHEMATICAL STRUCTURES (18CS36)

PREVIOUS YEARS QUESTION PAPERS

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17CS36

Third Semester B.E. Degree Examination, June/July 2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define tautology. Verify that $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology. **(06 Marks)**

b. If statement q has truth value 1, determine all truth value assignments for the primitive statements p, r, s for which the truth value of the statement :
$(q \to [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \to (\neg r \wedge q)]$ is 1. **(04 Marks)**

c. Establish the following logical equivalence :
i) $p \vee q \vee (\neg p \wedge \neg q \wedge r) \Leftrightarrow p \vee q \vee r$
ii) $[(\neg p \vee \neg q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$. **(10 Marks)**

OR

2 a. Establish the validity of following arguments :

i) $(\neg p \vee \neg q) \to (r \wedge s)$
$r \to t$
$\neg t$
$\therefore p$

ii) $u \to r$
$(r \wedge s) \to (p \vee t)$
$q \to (u \wedge s)$
$\neg t$
q
$\therefore p$ **(08 Marks)**

b. Let p(x), q(x) and r(x) be the following open statements :
$p(x) : x^2 - 7x + 10 = 0$ $q(x) : x^2 - 2x - 3 = 0$ $r(x) < 0$.
Determine truth or falsity of following statements, where universe is all integers. If a statement is false, provide a counter example.
i) $\forall x[p(x) \to \neg r(x)]$ ii) $\forall x[q(x) \to r(x)]$
iii) $\exists x[q(x) \to r(x)]$ iv) $\exists x [p(x) \to r(x)]$. **(08 Marks)**

c. Prove that for all integers 'k' and 'ℓ', if 'k' and 'ℓ' are both even, then $k + \ell$ is even and $k\ell$ is even by direct proof. **(04 Marks)**

Module-2

3 a. Define well ordering principle and prove the following by mathematical induction :
i) $1^2 + 3^2 + 5^2 + ----- + (2n-1)^2 = \dfrac{n(2n-1)(2n+1)}{3}$
ii) $1*3 + 2*4 + 3*5 + ----- + n(n+2) = \dfrac{n(n+1)(2n+7)}{6}$ **(12 Marks)**

b. Find the coefficients of :
i. x^9y^3 in the expansion of $(2x - 3y)^{12}$
ii. $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(08 Marks)**

OR

4 a. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in following situations,
   i. There is no restriction on the choice
   ii. Two particular persons will not attend separately
   iii. Two particular persons will not attend together. **(06 Marks)**

b. How many arrangements are there for all letters in word SOCIOLOGICAL? In how many of these arrangements all vowels are adjacent. **(06 Marks)**

c. For the Fibonacci sequence $F_0, F_1, F_2 \ldots$ prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. **(08 Marks)**

Module-3

5 a. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$.
   i. How many functions are there from A to B?
   ii. How many of these are one to one?
   iii. How many are onto?
   iv. How many functions are there from B to A?
   v. How many of these are onto?
   vi. How many are one to one? **(06 Marks)**

b. A computer operator is given a magnetic tape that contains 500,000 words of four or fewer lowercase letters. Can it be that the 500,000 words are all distinct? **(06 Marks)**

c. Let f, g, h : $R \to R$ where $f(x) = x^2$, $g(x) = x + 5$ and $h(x) = \sqrt{x^2+2}$. Show that (hog) of = ho(gof). **(08 Marks)**

OR

6 a. Let $A = \{1, 2, 3, 6, 9, 18\}$ and define R on A by xRy if "x divides y", Draw the Hasse diagram for the poset (A, R). Also write the matrix of relation. **(08 Marks)**

b. Consider Poset whose Hasse diagram is given below. Consider $B = \{3, 4, 5\}$. Find upper and lower bounds of B, least upper bound and greatest lower bound of B. **(04 Marks)**
(Ref. Fig.Q6(b)).

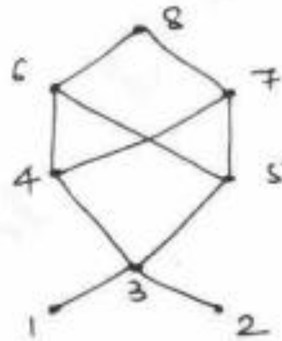

Fig.Q6(b)

c. Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$ and define R on A by (x_1, y_1) R (x_2, y_2) if $x_1 + y_1 = x_2 + y_2$.
   i. Verify that R is an equivalence relation on A
   ii. Determine equivalence classes $[(1, 3)]$, $[(2, 4)]$ and $[(1, 1)]$
   iii. Determine partition of A induced by R. **(08 Marks)**

Module-4

7 a. In how many ways can the 26 letters of English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(08 Marks)**

b. There are eight letters to eight different people to be placed in eight different addressed envelops. Find the number of ways of doing this so that atleast one letter gets to right person. **(04 Marks)**

c. Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party find that only one chair at each of five table T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs. **(08 Marks**

OR

8 a. Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30, 42, ... Hence find the general term of the sequence. **(10 Marks)**

b. If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 37$ satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$ for $n \geq 0$, determine the constants b and c and then solve the relation for a_n. **(10 Marks)**

Module-5

9 a. Merge sort the list –1, 7, 4, 11, 5, –8, 15, –3, –2, 6, 10, 3. **(06 Marks)**

b. Determine whether the following graphs are isomorphic or not. **(06 Marks)**

Fig.Q9(b)

c. Define the following with an example to each.
i) Simple graph ii) Complete graph iii) Regular graph iv) Spanning sub graph v) Induced subgraph vi) Complete Bipartite graph vii) Tree viii) Complement of graph. **(08 Marks)**

OR

10 a. Define trail, circuit, path, cycle. In the graph shown below determine : [Ref.Q10(a)]
i. a walk from b to d that is not a trail
ii. b-d trail that is not a path
iii. a path from b to d
iv. a closed walk from b to b that is not a circuit
v. a circuit from b to b that is not cycle
vi. a cycle form b to b. **(10 Marks)**

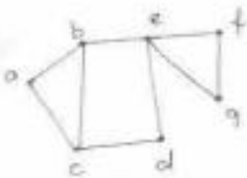

Fig.Q10(a)

b. Define optimal tree and construct an optimal tree for a given set of weights {4, 15, 25, 5, 8, 16}. Hence find the weight of optimal tree. **(06 Marks)**

c. Prove that in a graph. The sum of degrees of all vertices is an even number and is equal to twice the number of edges in the graph. **(04 Marks)**

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17CS36

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Define proposition, tautology, contradiction. Determine whether the following compound statement is a tautology or not.
$\{(p \vee q) \to r\} \leftrightarrow \{\neg r \to \neg(p \vee q)\}$ **(06 Marks)**

b. Using the laws of logic, show that $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$ **(07 Marks)**

c. Establish the validity of the following argument

$$\begin{array}{l} \forall x,\ p(x) \vee q(x) \\ \exists x,\ \neg p(x) \\ \forall x,\ \neg q(x) \vee r(x) \\ \underline{\forall x,\ s(x) \to \neg r(x)} \\ \therefore\ \exists x,\ \neg s(x) \end{array}$$

(07 Marks)

OR

2 a. Define converge, inverse and contra positive of a conditional. Find converse, inverse and contra positive of $\forall x,\ (x > 3) \to (x^2 > 9)$, where universal set is R. **(06 Marks)**

b. Test the validity of the following arguments:

i) If there is a strike by students, the exam will be postponed but the exam was not postponed.
―――――――――――
∴ there was no strike by students.

ii) If Ravi studies, then he will pass in DMS.
If Ravi doesn't play cricket, then he will study.
Ravi failed in DMS.
―――――――――――
∴ Ravi played cricket

(06 Marks)

c. Define dual of logical statement. Write the dual of the statement $(p \vee T_0) \wedge (q \vee F_0) \vee (r \wedge s \wedge T_0)$. **(02 Marks)**

d. Let $p(x) : x \geq 0$
$q(x) : x^2 \geq 0$ and $r(x) : x^2 - 3x - 4 = 0$
Then, for the universe completing of all real numbers, find the truth values of :
i) $\exists x\{p(x) \wedge q(x)\}$ ii) $\forall x\{p(x) \to q(x)\}$ iii) $\exists x\{p(x) \wedge r(x)\}$ **(06 Marks)**

Module-2

3 a. Prove that for any positive integer n, $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$, F_n denote the Fibonacci number. **(06 Marks)**

b. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000? **(07 Marks)**

c. Determine the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(07 Marks)**

OR

4 a. Prove by using principle of mathematical induction

$$\sum_{i=1}^{n} i \cdot 2^i = 2 + (n-1) \cdot 2^{n+1}$$

(06 Marks)

b. A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carries out if
   i) There are no restrictions
   ii) There must be six men and six women
   iii) There must be an even number of women. **(07 Marks)**

c. Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$ where $x_i \geq 0$, $1 \leq i \leq 4$. **(07 Marks)**

Module-3

5 a. If A = {1, 2, 3, 4, 5} and there are 6720 injective functions $f: A \to B$, what is $|B|$? **(03 Marks)**

b. Let m, n be positive integers with $1 < n \leq m$ then prove that,
   $s(m+1, n) = s(m, n-1) + ns(m, n)$ **(05 Marks)**

c. If $f: R \to R$ defined by $f(x) = x^2$, determine whether the function is one-to-one and whether it is onto. If it is not onto, find the range. **(06 Marks)**

d. Let A = {1, 2, 3, 4, 5} × {1, 2, 3, 4, 5} and define R on A by (x_1, y_1) R (x_2, y_2) if $x_1 + y_1 = x_2 + y_2$, verify that R is an equivalence relation on A. **(06 Marks)**

OR

6 a. If $f: R \to R$ defined by $f(x) = x^3$, determine whether f is invertible and if determine f^{-1}. **(05 Marks)**

b. Define the relation R for two lines ℓ_1 and ℓ_2 by ℓ_1 R ℓ_2 if ℓ_1 is perpendicular to ℓ_2. Determine whether the relation is reflexive, symmetric, antisymmetric or transitive. **(05 Marks)**

c. Let A = {1, 2, 3, 6, 9, 18} and R on A by xRy if x|y. Draw the Hasse diagram for the poset(A, R). **(05 Marks)**

d. For A = {1, 2, 3, 4}, let R{(1, 1) (1, 2) (2, 3) (3, 3) (3, 4)} be a relation on A. Draw the directed graph G on A that is associated with R. Do likewise for R^2, R^3. **(05 Marks)**

Module-4

7 a. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. How many derangements are there for 1, 2, 3, 4 and 5? **(07 Marks)**

c. Solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$, $n \geq 0$, $a_0 = 0$, $a_1 = 1$, $a_2 = 2$. **(07 Marks)**

OR

8 a. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, pun or byte occurs? **(06 Marks)**

b. Find the root polynomial for 3 × 3 board using the expansion formula. **(07 Marks)**

c. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **(07 Marks)**

Module-5

9 a. Show that the graphs Fig.Q9(a)(i) and (ii) are isomorphic.

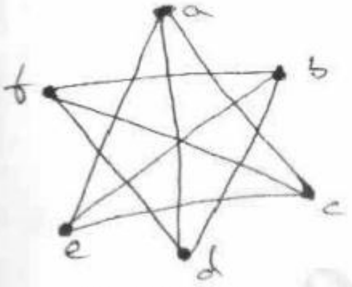

Fig.Q9(a)(i)

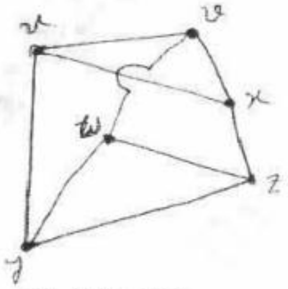

Fig.Q9(a)(ii) **(06 Marks)**

b. Let G = (V, E) be an undirected graph or multigraph with no isolated vertices. Then prove that G has an Euler circuit if and only if G is connected and every vertex in G has even degree. **(07 Marks)**

c. Construct an optimal prefix code for the symbols a, b, c, d, e, f, g, h, i, j that occur with respective frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3. **(07 Marks)**

OR

10 a. Let G = (V, E) be a connected undirected graph. What is the largest possible value for $|V|$ if $|E| = 19$ and $\deg(v) \geq 4$ for all $v \in V$? **(06 Marks)**

b. For every tree T = (V, E) if $|V| \geq 2$, then prove that T has atleast two pendant vertices. **(07 Marks)**

c. For the tree shown in Fig.Q10(c), list the vertices according to a preorder and a postorder traversal.

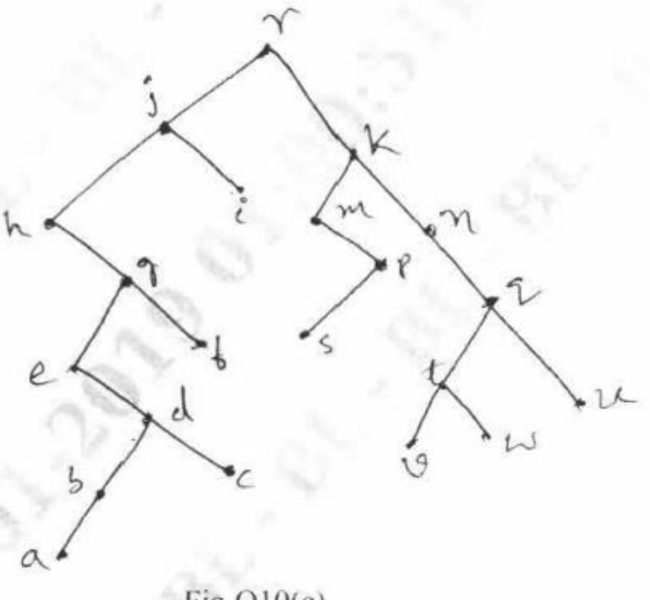

Fig.Q10(c) **(07 Marks)**

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15CS36

Third Semester B.E. Degree Examination, June/July 2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Simplify the switching network shown in Fig Q1(a)

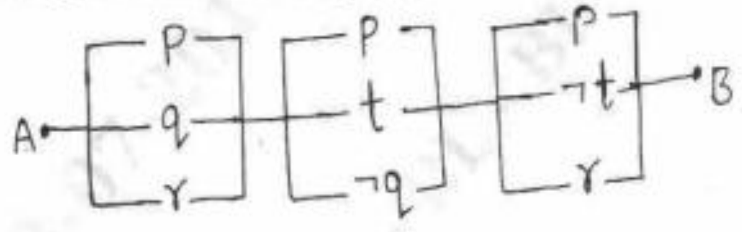

Fig Q1(a) **(08 Marks)**

b. Give a direct proof of the statement "If n is an odd integer then n^2 is also an odd integer". **(04 Marks)**

c. Let p(x), q(x) and r(x) be open statements that are defined for the given universe. Show that the argument

$\forall x, [p(x) \rightarrow q(x)]$

$\underline{\forall x, [q(x) \rightarrow r(x)]}$

$\therefore \exists x, [p(x) \rightarrow r(x)]$ is valid **(04 Marks)**

OR

2 a. Define tautology, prove that for any proposition p, q, r the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow q)$ is a tautology using truth table. **(05 Marks)**

b. Show that RVS follows logically form the premises CVD, $CVD \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ and $(A \wedge \neg B) \rightarrow (RVS)$. **(04 Marks)**

c. Using rules of inference shows that the following argument is valid.

$\forall x, [p(x) \vee q(x)] \wedge \exists x, \neg p(x) \wedge$

$\forall x, [\neg q(x) \vee r(x)] \wedge \forall x, [s(x) \rightarrow \neg r(x)]$

$\therefore \exists x, \neg S(x)$ **(07 Marks)**

Module-2

3 a. Prove by mathematical induction that, for all integers $n \geq 1$, $1 + 2 + 3 + \ldots + n = \frac{1}{2} n(n+1)$. **(06 Marks)**

b. The Fibonacci numbers are defined recursively by $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Evaluate F_2 to F_{10}. **(04 Marks)**

c. In the word S, O, C, I, O, L, O, G, I, C, A, L.

i) How many arrangements are there for all letters?

ii) In how many of these arrangements all vowels are adjacent? **(06 Marks)**

OR

4 a. Obtain the recursive definition for the sequence $\{a_n\}$ in each of the following cases.
(i) $a_n = 5n$ (ii) $a_n = 6^n$ (iii) $a_n = n^2$ **(06 Marks)**

b. Find the coefficient of
i) $x^9 y^3$ in the expansion fo $(2x - 3y)^{12}$
ii) x^{12} in the expansion of $x^3 (1 - 2x)^{10}$ **(04 Marks)**

c. A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 blank spaces between the symbols, with atleast 3 spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message? **(06 Marks)**

Module-3

5 a. Let $f: R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$$

determine $f(0)$, $f(-1)$, $f^{-1}(0)$, $f^{-1}(+3)$, $f^{-1}([-5, 5])$ **(08 Marks)**

b. Define an equivalence relation. Write the partial order relation for the positive divisors of 36 and write its Hasse diagram (HASSE). **(08 Marks)**

OR

6 a. Consider the function $f: R \to R$ defined by $f(x) = 2x + 5$. Let a function $g: R \to R$ be defined by $g(x) = \frac{1}{2}(x - 5)$. Prove that g is an inverse of f. **(03 Marks)**

b. State Pigeonhole principle. Let ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that atleast two of their points are such that the distance between them is less than ½ cm. **(05 Marks)**

c. If $A = \{1, 2, 3, 4\}$, R and S are relations on A defined by $R = \{(1, 2), (1, 3), (2, 4), (4, 4)\}$ $S = \{(1, 1), (1, 2), (1,3), (1, 4), (2, 3), (2, 4)\}$ find RoS, SoR, R^2, S^2 and write down their matrices. **(08 Marks)**

Module-4

7 a. Find the number of derangements of 1, 2, 3, 4 list all such derangements. **(04 Marks)**

b. Determine the number of integers between 1 and 300 (inclusive) which are divisible by exactly 2 of 5, 6, 8. **(06 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day? **(06 Marks)**

OR

8 a. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for 5 classes C_1, C_2, C_3, C_4, C_5 one teacher for each class T_1 and T_2 donot wish become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5 and T_5 for C_3 or C_4 or C_5. In how many ways can teachers be assigned the work (without displeasing any teacher)? **(08 Marks)**

b. Solve the recurrence relation,
$a_n = 2(a_{n-1} - a_{n-2})$, where $n \geq 2$ and $a_0 = 1$, $a_1 = 2$. **(08 Marks)**

Module-5

9 a. Prove that the undirected graph G = (V, E) has an Euler circuit if and only if G is connected and every vertex in G has even degree. **(08 Marks)**

b. Define binary rooted tree and Balanced tree. Draw all the spanning trees of the graph shown in Fig 9(b)

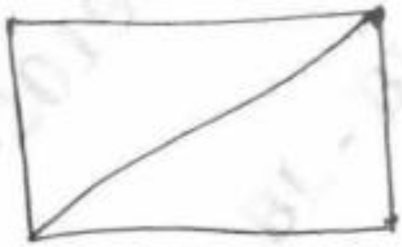

Fig Q9(b) **(08 Marks)**

OR

10 a. Define, with an example for each Regular graph, complement of a graph, Euler trail and Euler circuit and complete graph. **(10 Marks)**

b. Apply Merge sort to the list
6, 2, 7, 3, 4, 9, 5, 1, 8 **(06 Marks)**

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CBCS SCHEME

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15CS36

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define Tautology. Verify the following compound proposition is a tautology or not : $\{(p \vee q) \to r\} \leftrightarrow \{\sim r \to \sim (p \vee q)\}$. **(04 Marks)**

b. Check whether the following argument is valid or not :
If I study, then I will not fail in exam.
If I do not watch TV in the evenings, then I will study.
I failed in exam.
$\therefore$ I must have watched TV in the evenings. **(04 Marks)**

c. Define : i) open sentence ii) quantifiers. Write the following proposition in symbolic form and find its negation : "All integers are rational numbers and some rational numbers are integers". **(04 Marks)**

d. Give a direct proof of the statement, "For all integers K and ℓ, if K and ℓ are both even then $K + \ell$ is even and $K\ell$ is even". **(04 Marks)**

OR

2 a. Define converse, inverse and contra positive of an implication. Hence find converse, inverse and contra positive for "$\forall x, (x > 3) \to (x^2 > 9)$" where universal set is the set of real numbers R. **(04 Marks)**

b. Using the laws of logic, prove the following logical equivalence : $[(\sim p \vee \sim q) \wedge (F_0 \vee p) \wedge P] \Leftrightarrow p \wedge \sim q$. **(04 Marks)**

c. What are bound variables and free variables. Identify the same in each of the following expressions :
i) $\forall y, \exists z \{\cos(x + y) = \sin(z - x)\}$
ii) $\exists x, \exists y \{(x^2 - y^2) = z\}$. **(04 Marks)**

d. Verify the validity of the following argument : If a triangle has two equal sides, then it is isosceles. If a triangle is isosceles, then it has two equal angles. The triangle ΔABC does not have two equal angles. $\therefore \Delta ABC$ does not have two equal sides. **(04 Marks)**

Module-2

3 a. Prove by mathematical induction $1.3 + 2.4 + 3.5 + \cdots + n(n+2) = \dfrac{n(n+1)(2n+7)}{6}$. **(04 Marks)**

b. Give a recursive definition for each of the following integer sequence :
i) $a_n = 7n$ ii) $a_n = 2 - (-1)^n$ for $n \in z^+$. **(04 Marks)**

c. How many positive integers can be formed by using the digits 3, 4, 4, 5, 5, 6, 7 to exceed 5,000,000? **(04 Marks)**

d. In how many ways can we distribute seven apples and six oranges among four children so that each child receives at least one apple? **(04 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

4 a. If F_0, F_1, F_2, - - - - are Fibonacci numbers, then prove by induction $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$. **(04 Marks)**

b. A sequence $\{a_n\}$ is defined recursively as $a_1 = 7$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$. Find a_n in explicit form. **(04 Marks)**

c. Find the number of arrangements of all the letters in the word "TALLAHASSEE". How many of these arrangements have no adjacent A's? **(04 Marks)**

d. Find the coefficient of $w^3x^2yz^2$ in the expansion of $(2w - x + 3y - 2z)^8$. **(04 Marks)**

Module-3

5 a. Define Cartesian product of two sets. For any three non-empty sets A, B and C. Prove that $A \times (B - C) = (A \times B) - (A \times C)$. **(04 Marks)**

b. Let f and g be two functions form R to R defined by $f(x) = 2x + 5$ and $g(x) = \frac{x-5}{2}$. Show that f and g are invertible to each other. **(04 Marks)**

c. Define partition of a set. If R is a relation defined on $A = \{1, 2, 3, 4\}$ by $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$, determine the partition induced by R. **(04 Marks)**

d. Let $A = \{a, b, c\}$, $B = P(A)$ where P(A) is the power set of A. Let R be a subset relation on A. Show that (B, R) is a POSET and draw its Hasse diagram. **(04 Marks**

OR

6 a. Let R be an equivalence relation on set A and $a, b \in A$. Then prove the following are equivalent :

i) $a \in [a]$

ii) a R b iff $[a] = [b]$

iii) if $[a] \cap [b] \neq \phi$ then $[a] = [b]$. **(04 Marks)**

b. Prove that a function $f : A \rightarrow B$ is invertible iff it is one – one and onto. **(04 Marks)**

c. State Pigeonhole principle. Show that if any seven numbers from 1 to 12 are chosen, then two of them will add to 13. **(04 Marks)**

d. Show that the set of positive divisors of 36 is a POSET and draw its Hasse diagram. Hence find its i) least element ii) greatest element. **(04 Marks**

Module-4

7 a. Out of 30 students in a hostel, 15 study history, 8 study economics and 6 study geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(04 Marks)**

b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all these derangements. **(04 Marks)**

c. Find the rook polynomial for the following board [refer Fig.Q7(c)] :

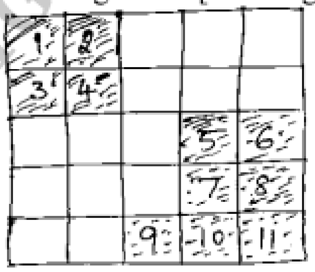

Fig. Q7(c) **(04 Marks)**

d. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(04 Marks)**

OR

8 a. Determine the number of integers between 1 and 300 (inclusive) which are
i) divisible by exactly two of 5, 6, 8 ii) divisible by at least two of 5, 6, 8. **(04 Marks)**

b. In how many ways can be integers 1, 2, - - - -, 10 be arranged in a line so that no even integer is in its natural place. **(04 Marks)**

c. An apple, a banana, a mango and an ornage are to be distributed to four boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have apple, the boy B_3 does not want banana or mango, B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(04 Marks)**

d. Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$ given that $F_0 = 0$, $F_1 = 1$. **(04 Marks)**

Module-5

9 a. Define the following with an example for each :
i) Complete graph ii) regular graph iii) bipartite graph iv) complete bipartite graph. **(04 Marks)**

b. Define isomorphism of two graphs. Verify the following graphs are isomorphic or not : [Refer Fig.Q9(b)] **(04 Marks)**

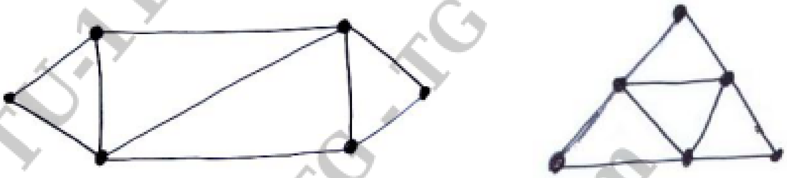

Fig.Q9(b)

c. Show that a tree with n vertices has n – 1 edges. **(04 Marks)**

d. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(04 Marks)**

OR

10 a. Explain Konigsberg bridge problem. **(04 Marks)**

b. Define the following with an example :
i) subgraph ii) spanning subgraph
iii) induced subgraph iv) edge–disjoint and vertex – disjoint subgraphs. **(04 Marks)**

c. If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and one vertex of degree 5, find the number of leaves in T. **(04 Marks)**

d. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. **(04 Marks)**

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CBCS SCHEME

USN

15CS36

Third Semester B.E. Degree Examination, June/July 2018

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Prove that for any propositions p, q, r the compound proposition :
$\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology. **(06 Marks)**

b. Prove the following logical equivalence using the laws of logic:
$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$. **(05 Marks)**

c. Prove the following logical equivalence using the laws of logic:
$[\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. **(05 Marks)**

OR

2 a. Prove the validity of the arguments using rule of inference.

$$\begin{array}{l} (\neg p \vee \neg q) \to (r \wedge s) \\ r \to t \\ \neg t \\ \hline \therefore p \end{array}$$

(05 Marks)

b. Test the validity of the arguments using rule of inference.

$$\begin{array}{l} (\neg p \vee q) \to r \\ r \to (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \to \neg t \\ \hline \therefore p \end{array}$$

(05 Marks)

c. Find whether the following argument is valid:

No Engineering student of 1^{st} or 2^{nd} semester studies logic
Anil is an Engineering student who studies logic

$\therefore$ Anil is not in second semester. **(06 Marks)**

Module-2

3 a. Prove by mathematical induction that :
$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$. **(05 Marks)**

b. A sequence $\{C_n\}$ is defined recursively by ,
$C_n = 3C_{n-1} - 2C_{n-2}$ for all $n \geq 3$ with $C_1 = 5$ and $C_2 = 3$ as the initial conditions, show that $C_n = -2^n + 7$. **(06 Marks)**

c. Determine the coefficient of xyz^2 in the expansion of $(2x - y - z)^4$. **(05 Marks)**

OR

4 a. A certain question paper contains two parts A and B, each containing 4 questions. How many different ways a student can answer 5 questions by selecting atleast 2 questions from each part? **(05 Marks)**

b. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(06 Marks)**

c. How many numbers greater than 1000000 can be formed by using the digits 1, 2, 2, 2, 4, 4, 0? **(05 Marks)**

Module-3

5 a. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$$

Determine $f^{-1}(0), f^{-1}(1), f^{-1}(-1), f^{-1}(3), f^{-1}(-3), f^{-1}(-6)$, **(06 Marks)**

b. Evaluate S(5, 4). **(05 Marks)**

c. Let f, g, h be the function form R to R defined by $f(x) = x + 2$, $g(x) = x - 2$, $h(x) = 3x$ for all $x \in R$. Find gof, fog, fof, hog, foh. **(05 Marks)**

OR

6 a. Let 'S' be the set of all non-zero integers and $A = S \times S$ on A, define the relation R by $(a, b)R(c, d)$ if and only if $ad = bc$. Show that 'R' is an equivalence relation. **(06 Marks)**

b. Draw the Hasse diagram representing the positive divisors of 36. **(06 Marks)**

c. Let A = {a, b, c, d, e}. Consider the partition P = {{a, b}, {c, d}, {e}} of A. Find the equivalence relation inducing this partition. **(04 Marks)**

Module-4

7 a. In a survey of 260 college students, the following data were obtained. 64 had taken mathematics course, 94 had taken CS course, 58 had taken EC course, 28 had taken both Mathematics and EC course, 26 had taken both Mathematics and CS course, 22 had taken both CS and EC course, and 14 had taken all three types of course. Determine how many of these students had taken none of the three subjects. **(05 Marks)**

b. Find the rook polynomial for the 3×3 board using expansion formula. **(06 Marks)**

c. Solve the recurrence relation :

$a_n + a_{n-1} - 6a_{n-2} = 0 \quad n \ge 2$, given $a_0 = -1$ and $a_1 = 8$. **(05 Marks)**

OR

8 a. An apple, a banana, a mango and an orange are to be distributed among 4 boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have an apple, the boy B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased. **(06 Marks)**

b. How many permutation of 1, 2, 3, 4, 5, 6, 7, 8 are not derangements? **(04 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

Module-5

9 a. Define isomorphism. Show that the following graph are isomorphic to each other. Refer Fig.Q9(a). **(06 Marks)**

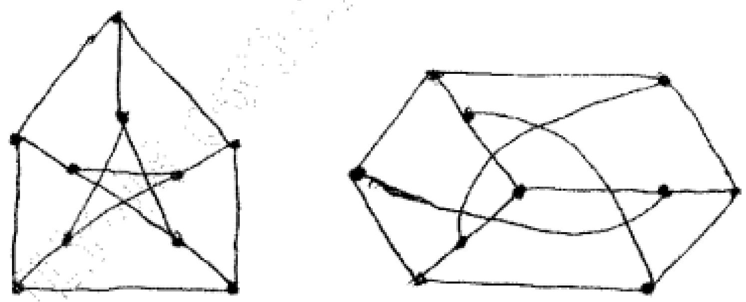

Fig.Q9(a)

b. "A tree with 'n' vertices is having 'n – 1' edges". Prove the given statement. **(05 Marks)**

c. Define complete graph, general graph and Bipartite graph with example for each. **(05 Marks)**

OR

10 a. For a graph with 'n' vertices and 'm' edges, if 'δ' is minimum, 'Δ' is maximum of the degree of vertices. Show that :

$$\delta \le \frac{2m}{n} \le \Delta .$$ **(05 Marks)**

b. Obtain the optimal prefix code for the message "ROAD IS GOOD". Indicate the code. **(06 Marks)**

c. Apply the merge sort to the following given list of element.

$\{-1, 0, 2, -2, 3, 6, -3, 5, 1, 4\}$. **(05 Marks)**

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4

USN

Third Semester B.E. Degree Examination, Dec.2017/Jan.2018
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Prove that for any three propositions p, q, r $[P \to (q \wedge r)] \Leftrightarrow [(p \to q) \wedge (p \to r)]$. Using truth table. **(05 Marks)**

b. Establish the validity of the argument :

$p \to q$
$q \to (r \wedge s)$
$\neg r \vee (\neg t \vee u)$
$\underline{p \wedge t}$
$\therefore u$ **(06 Marks)**

c. Prove that for all integers 'k' and 'ℓ', if 'k' and 'ℓ' are both odd, then $k + \ell$ is even and $k\ell$ is odd by direct proof. **(05 Marks)**

OR

2 a. Determine the truth value of each of the following quantified statements; the universe being the set of all non - zero integers. **(05 Marks)**

i) $\exists x, \exists y\, [xy = 1]$
ii) $\exists x, \forall y\, [xy = 1]$
iii) $\forall x, \exists y, [xy = 1]$
iv) $\exists x, \exists y\, [(2x + y = 5) \wedge (x - 3y = -8)]$.
v) $\exists x, \exists y\, [(3x - y = 17) \wedge (2x + 4y = 3)]$. **(06 Marks)**

b. Find whether the following arguments are valid or not for which the universe is set of all triangles. In triangle XYZ, there is no pair of angles of equal measure. If the triangle has two sides of equal length, then it is isosceles. If the triangle is isosceles, then it has two angles of equal measure. Therefore triangle XYZ has no two sides of equal length. **(05 Marks)**

c. If a proposition has truth value 1, determine all truth value assignments for the primitive propositions p, r, s for which the truth value of following compound proposition is 1.

$[q \to \{(\neg p \vee r) \wedge \neg s\}] \wedge \{\neg s \to (\neg r \wedge q)\}$. **(05 Marks)**

Module-2

3 a. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(05 Marks)**

b. For the Fibonacci sequence F_0, F_1, F_2 - - - - prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. **(06 Marks)**

c. Find the coefficient of :

i) x^9y^3 in the expansion $(2x - 3y)^{12}$
ii) x^{12} in the expansion $x^3(1 - 2x)^{10}$. **(05 Marks)**

OR

4 a. By mathematical induction. Prove that, for every positive integer n, the number $A_n = 5^n + 2.3^{n-1} + 1$ is a multiple of 8. **(05 Marks)**

b. How many positive integers 'n' can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want 'n' to exceed 5,000,000. **(06 Marks)**

c. A certain question paper contains three parts A, B, C with four questions in part A, five questions in part B and six questions in part C. It is required to answer seven questions selecting atleast two questions from each part. In how many ways can a student select his seven questions for answering? **(05 Marks)**

Module-3

5 a. Let $f : R \to R$ be defined by $f(x) = \begin{cases} 3x-5, & \text{for } x > 0 \\ -3x+1, & \text{for } x \le 0 \end{cases}$

i) Determine $f\left(\frac{5}{3}\right)$, $f^{-1}(3)$, $f^{-1}([-5, 5])$.

ii) Also prove that if 30 dictionaries contain a total of 61, 327 pages, then atleast one of the dictionary must have atleast 2045 pages. **(05 Marks)**

b. Prove that if $f : A \to B$ and $g : B \to C$ are invertible function then $g \text{ o} f : A \to C$ is an invertible function and $(gof)^{-1} = f^{-1}og^{-1}$. **(06 Marks)**

c. Let A = {1, 2, 3, 4, 5}. Define a relation R on A × A by $(x_1, y_1)\, R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Determine whether R is an equivalence relation on A × A

ii) Determine equivalence class [(1, 2)], [(2, 5)]. **(05 Marks)**

OR

6 a. Let f and g be functions from R to R defined by f(x) = ax + b and $g(x) = 1 - x + x^2$. If $(g\,o\,f)(x) = 9x^2 - 9x + 3$. Determined a, b. **(05 Marks)**

b. Let A = {1, 2, 3, 4, 6, 12}. On A define the relation R by aRb if and only if 'a' divides 'b' i) prove that R is a partial order on A ii) draw the Hasse diagram iii) write down the matrix of relation. **(06 Marks)**

c. Consider the Poset whose Hasse diagram is given below. Consider B = {3, 4, 5}. Refer Fig.Q6(c). Find :

i) All upper bounds of B
ii) All lower bounds of B
iii) The least upper bound of B
iv) The greatest lower bound of B
v) Is this a Lattice? **(05 Marks)**

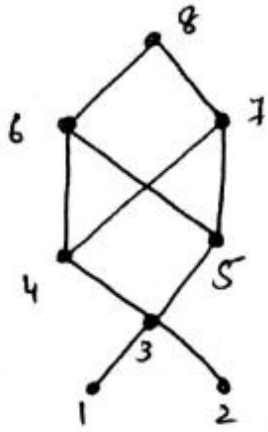

Fig.Q6(c)

Module-4

7 a. Out of 30 students in a hostel; 15 study history 8 study economics and 6 study geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(05 Marks)**

b. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes C_1, C_2, C_3, C_4, C_5, one teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5 and T_5 for C_3 or C_4 or C_5. In how many ways can the teachers be assigned work without displeasing any teacher. **(06 Marks)**

c. Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 0$ form $n \geq 2$. **(05 Marks)**

OR

8 a. Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$. **(05 Marks)**

b. Let a_n denote the number of n-letter sequences that can be formed using the letters A, B and C such that non terminal A has to be immediately followed by a B. Find the recurrence relation for a_n and solve it. **(06 Marks)**

c. Find the number of permutations of English letters which contain exactly two of the pattern car, dog, pun, byte. **(05 Marks)**

Module-5

9 a. Discuss Konigsberg bridge problem. **(05 Marks)**

b. Let G = G(V, E) be a simple graph with m edges and 'n' vertices. Then prove that :

i) $m \leq \frac{1}{2}n(n-1)$

ii) For a complete graph k_n, $m = \frac{1}{2}n(n-1)$ edges

iii) How many vertices and edges are there for $K_{4,7}$ and $K_{7,11}$. **(06 Marks)**

c. Merge sort the list –1, 7, 4, 11, 5, –8, 15, –3, –2, 6, 10, 3. **(05 Marks)**

OR

10 a. Prove that a tree with 'n' vertices has n – 1 edges. **(05 Marks)**

b. Obtain an optimal prefix code for the message LETTER RECEIVED indicate the code and weight. **(06 Marks)**

c. Determine whether the following graphs are isomorphic or not. **(05 Marks)**

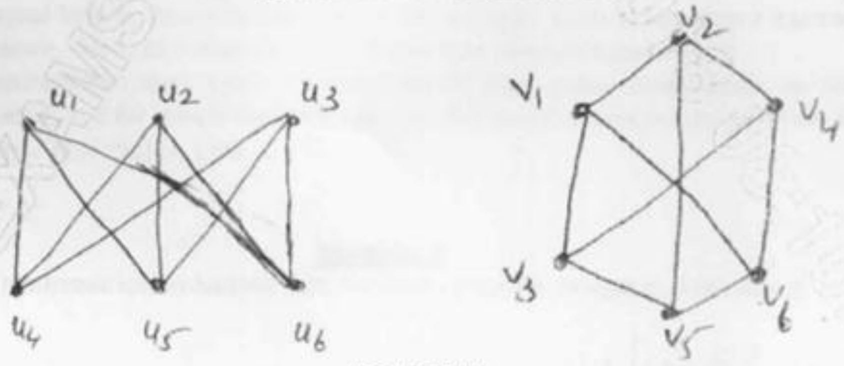

Fig.Q10(c)

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CBCS Scheme

USN | | | | | | | | | |

15CS36

Third Semester B.E. Degree Examination, June/July 2017

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define the following with an example for each
   i) Proposition
   ii) Tautology
   iii) Contradiction
   iv) Dual of statement. **(06 Marks)**

b. Establish the validity of the following argument using rules of inference. If the band could not play rock music or the refreshments were not served on time, then the new year party could have been cancelled and Alica would have been angry. If the party were cancelled, then refunds would have to be made. No refunds were made, therefore the band could play rock music. **(05 Marks)**

c. Determine the truth value of the following statements if the universe comprises all nonzero integers :
   i) $\exists x \exists y[xy = 2]$
   ii) $\exists x \forall y[xy = 2]$
   iii) $\forall x \exists y[xy = 2]$
   iv) $\exists x \exists y[(3x + y = 8) \wedge (2x - y) = 7]$
   v) $\exists x \exists y[(4x + 2y = 3) \wedge (x - y = 1)]$ **(05 Marks)**

OR

2 a. Find the possible truth values for p, q and r if
   i) $p \rightarrow (q \vee r)$ – FALSE
   ii) $p \wedge (q \rightarrow r)$ – TRUE. **(05 Marks)**

b. Show that $(p \wedge (p \rightarrow q)) \rightarrow q$ is independent of its components. **(06 Marks)**

c. Give a direct proof for each of the following :
   i) For all integers k and ℓ, if k and ℓ are both even, then k + ℓ is even
   ii) For all integers k and ℓ, if k and ℓ are both even, then k * ℓ is even. **(05 Marks)**

Module-2

3 a. Prove by mathematical induction, for every positive integer 8 divides $5^n + 2 \cdot 3^{n-1} + 1$. **(06 Marks)**

b. Assuming PASCAL language is case insensitive, an identifier consists of a single letter followed by upto seven symbols which may be letters or digits (26 letters, 10 digits). There are 36 reserved words. How many distinct identifiers are possible in this version of PASCAL? **(05 Marks)**

c. Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(05 Marks)**

OR

4 a. Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(05 Marks)**

b. Lucas numbers are defined recursively as $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$. If Fi^s are fibonacci numbers and Li^s are the Lucas numbers, prove that $L_n = F_{n-1} + F_{n+1}$ for all positive integers n. **(05 Marks)**

c. Find the number of distinct terms in the expansion of $(w + x + y + z)^{12}$. **(06 Marks)**

Module-3

5 a. Let A = {a, b, c, d} and B = {1, 2, 3, 4, 5, 6}.

i) How many functions are there from A to B? How many of these are one-to-one? How many are onto?

ii) How many functions are there from B to A? How many of these are one-to-one? How many are onto? **(06 Marks)**

b. Prove that if $f : A \rightarrow B$, $g : B \rightarrow C$ are invertible functions, then $g \circ f : A \rightarrow C$ is invertible and $(gof)^{-1} = f^{-1}og^{-1}$. **(06 Marks)**

c. For the Hasse diagram, given in Fig. Q5(c), write i) maximal ii) minimal iii) greatest and iv) least element (s). **(04 Marks)**

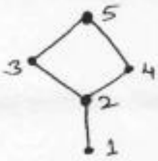

Fig. Q5(c)

OR

6 a. Let $f, g : z^+ \rightarrow z^+$, where for all $x \in z^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x - 1\}$.

i) What is the range of f?

ii) Is f a onto function?

iii) Is f one-to-one?

iv) What is the range of g?

v) Is g an onto function? **(05 Marks)**

b. If $f : A \rightarrow B$ and $B_1, B_2 \leq B$, then prove the following :

i) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$

ii) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$

iii) $f^{-1}(\overline{B}_1) = \overline{f^{-1}(B_1)}$ **(06 Marks)**

c. Let A = {1, 2, 3, 4}, R = {(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)} be the relation on A. Determine whether the relation R is reflexive, irreflexive, symmetric, antisymmetric or transitive. **(05 Marks)**

Module-4

7 a. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(05 Marks)**

b. Describe the expansion formula for rook polynomials. Find the rook polynomial for 3×3 board using the expansion formula. **(05 Marks)**

c. Solve the recurrence relation $b_n = bD_{n-1} - b^2D_{n-2}$, $n \geq 3$ given $D_1 = b > 0$ and $D_2 = 0$. **(06 Marks)**

OR

8 a. In how many ways can we arrange the letters in the CORRESPONDENTS so that ;
   i) There is no pair of consecutive identical letters?
   ii) There are exactly two pairs of consecutive identical letters
   iii) There are atleast 3 pairs of consecutive identical letters **(06 Marks)**

b. Find the recurrence relation and the initial conditions for the sequence 0, 2, 6, 12, 20, 30, 42 ...... Hence find the general term of the sequence. **(05 Marks)**

c. Find the general solution of the equation $S(k) + 3S(k-1) - 4S(k-2) = 4^k$. **(05 Marks)**

Module-5

9 a. Define the following with an example
   i) Simple graph
   ii) Regular graph
   iii) Subgraph
   iv) Maximal subgraph
   v) Induced subgraph. **(05 Marks)**

b. Show that there exists no simple graphs corresponding to the following degree sequences
   i) 0, 2, 2, 3, 4
   ii) 1, 1, 2, 3
   iii) 2, 3, 3, 4, 5, 6
   iv) 2, 2, 4, 6. **(04 Marks)**

c. Let T = (V, E) be a complete m-ary tree with $|V| = n$. If T has ℓ leaves and i internal vertices, then prove the following :
   i) $n = m \cdot i + 1$
   ii) $\ell = (m-1)i + 1$
   iii) $i = \dfrac{(\ell - 1)}{(m-1)} = \dfrac{(n-1)}{m}$ **(07 Marks)**

OR

10 a. In the graph shown in Fig. Q10(a). Determine
   i) a walk from b to d that is not a trail
   ii) b – d trail that is not a path
   iii) a path from b to d
   iv) a closed walk from b to b that is not a circuit
   v) a circuit from b to b that is not a cycle
   vi) a cycle from b to b **(06 Marks)**

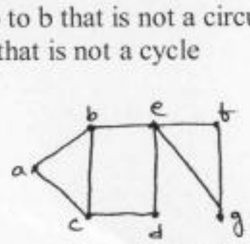

Fig. Q10(a)

b. Determine the order $|V|$ of the graph G = (V, E) in the following cases
   i) G is cubic graph with 9 edges
   ii) G is regular with 15 edges
   iii) G has 10 edges with 2 vertices of degree 4 and all other of degree 3. **(06 Marks)**

c. Obtain the optimal prefix code for the string ROAD IS GOOD. **(04 Marks)**

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CBCS Scheme

USN | | | | | | | | | | |

15CS36

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Let p, q and r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound proposition
i) $(p \wedge q) \rightarrow r$ ii) $p \rightarrow (q \wedge r)$ iii) $p \wedge (r \rightarrow q)$ iv) $p \rightarrow (q \rightarrow (\neg r))$ **(04 Marks)**

b. Define tautology. Prove that for any propositions p, q, r the compound proposition $[(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}] \rightarrow r$ is tautology. **(04 Marks)**

c. Establish the validity of the following argument

$$\begin{array}{l} \forall x, [p(x) \vee q(x)] \\ \exists x, \neg p(x) \\ \forall x, [\neg q(x) \vee r(x)] \\ \underline{\forall x, [s(x) \rightarrow \neg r(x)]} \\ \therefore \quad \exists x \ \neg s(x) \end{array}$$

(04 Marks)

d. Give i) direct proof and ii) proof by contradiction for the following statement. "If 'n' is an odd integer, then n+9 is an even integer". **(04 Marks)**

OR

2 a. Define dual of a logical statement. Verify the principle of duality for the following logical equivalence $[\sim (p \wedge q) \rightarrow \sim p \vee (\sim p \vee q)] \Leftrightarrow (\sim p \vee q)$. **(04 Marks)**

b. Prove the following by using laws of logic
i) $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$
ii) $[\sim p \wedge (\sim q \vee r)] \vee [(q \wedge r) \vee (p \wedge q)] \Leftrightarrow r$. **(04 Marks)**

c. Establish the validity of the following argument using the rules of inference:
$[p \wedge (p \rightarrow q) \wedge (s \vee t) \wedge (r \rightarrow \sim q)] \rightarrow (s \vee t)$ **(04 Marks)**

d. Define i) open sentence ii) quantifiers. For the following statements, the universe comprises all non-zero integers. Determine the truth values of each statement :
i) $\exists x, \exists y\, (xy = 1)$ ii) $\exists x, \forall y\, (xy = 1)$ iii) $\forall x, \exists y\, (xy = 1)$. **(04 Marks)**

Module-2

3 a. By mathematical induction, prove that

$$1^2 + 3^2 + 5^2 \ldots\ldots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}.$$

(05 Marks)

b. For the Fibonacci sequence show that **(05 Marks)**

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

c. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations : i) There is no restriction on the choice ii) Two particular persons will not attend separately iii) Two particular persons will not attend together. **(06 Marks)**

OR

4 a. Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and /or 7's. **(04 Marks)**

b. Find an explicit definition of the sequence defined recursively by $a_1 = 7$, $a_n = 2a_{n-1}+1$ for $n \geq 2$. **(04 Marks)**

c. i) How many arrangements are there for all letters in the word SOCIOLOGICAL?
ii) In how many of these arrangements A and G are adjacent? In how many of these arrangements all the vowels are adjacent? **(04 Marks)**

d. Find the coefficient of i) x^9y^3 in the expansion of $(2x - 3y)^{12}$ ii) $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(04 Marks)**

Module-3

5 a. Let a function $f : R \rightarrow R$ be defined by $f(x) = x^2 + 1$. Find the images of $A_1=\{2, 3\}$, $A_2 = \{-2, 0, 3\}$, $A_3 = (0, 1)$ and $A_4 = [-6, 3]$. **(04 Marks)**

b. ABC is an equilateral triangle whose sides are of length one cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½ cm. **(04 Marks)**

c. Let f, g, h be functions from z to z defined by $f(x) = x - 1$, $g(x) = 3x$

and $h(x) = \begin{cases} 0 & \text{if x is even} \\ 1 & \text{if x is added} \end{cases}$.

Determine $(fo(goh))(x)$ and $((fog)oh)(x)$ and verify that $fo(goh) = (fog)oh$. **(04 Marks)**

d. For A = {a, b, c, d, e} the Hasse diagram for the Poset (A, R) is as shown in Fig Q5(d). Determine the relation matrix for R and Construct the digraph for R **(04 Marks)**

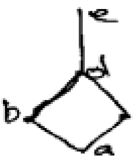

Fig Q5(d)

OR

6 a. Let A = {1, 2, 3} and B = {2, 4, 5}. Determine the
i) Number of binary relations on A.
ii) Number of relations from A to B that contain (1, 2) and (1, 5)
iii) Number of relations from A, B that contain exactly five ordered pairs
iv) Number of binary relations on A that contains at least seven ordered pairs. **(04 Marks)**

b. Let A = B = R be the set of the real numbers, the functions $f : A \rightarrow B$ and $g: B \rightarrow A$ be defined by $f(x) = 2x^3 - 1$, $\forall x \in A$; $g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3}$ $\forall y \in B$. Show that each of f and g is the inverse of the other. **(04 Marks)**

c. Define a relation R on A×A by $(x_1, y_1) R (x_2, y_2)$ iff $x_1+y_1 = x_2+y_2$, where A ={1, 2, 3, 4, 5}.
i) Verify that R is an equivalence relation on A×A.
ii) Determine the equivalence classes [(1, 3)] and [(2, 4)]. **(04 Marks)**

d. Consider the Hasse diagram of a POSET (A, R) given in Fig Q6(d). If B = {c, d, e} find all upper bounds, lower bounds, the least upper bound and the greatest lower bound of B. **(04 Marks)**

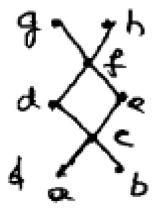

Fig Q6(d)

Module-4

7 a. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3, or 5. **(04 Marks)**

b. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(04 Marks)**

c. A girl student has Sarees of 5 different colors, blue, green red, white and yellow. On Monday she does not wear green, on Tuesdays blue or red, on Wednesday blue or green, on Thursday red or yellow; on Friday red. In how many ways can she dress without repeating a color during a week (from Monday to Friday)? **(04 Marks)**

d. The number of affected files in a system 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(04 Marks)**

OR

8 a. In how many ways can one arrange the letters in the word CORRESPONDENTS so that
   i) There is no pair of consecutive identical letters?
   ii) There are exactly two pairs of consecutive identical letters? **(06 Marks)**

b. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3, and B4. The boys B_1 and B_2 do not wish to have apple, the boy, B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(05 Marks)**

c. Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$ given that $a_1 = 5$ and $a_2 = 3$. **(05 Marks)**

Module-5

9 a. Define :
   i) Bipartite graph
   ii) Complete bipartite graph
   iii) Regular graph
   iv) Connected graph with an example. **(04 Marks)**

b. Define isomorphism. Verify the two graphs are isomorphic **(04 Marks)**

Fig Q9(b)

c. Show that a tree with n vertices has n-1 edges. **(04 Marks)**

d. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. **(04 Marks)**

OR

10 a. Determine the order $|V|$ of the graph G = (V, E) in
   i) G is a cubic graph with 9 edges
   ii) G is regular with 15 edges
   iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3. **(04 Marks)**

b. Prove that in a graph
   i) The sum of the degrees of all the vertices is an even number and is equal to twice the number of edges in the graph.
   ii) The number of vertices of odd degrees is even. **(04 Marks)**

c. Discuss the solution of Konigsberg bridge problem. **(04 Marks)**

d. Define optimal tree and construct an optimal tree for a given set of weights {4, 15, 25, 5, 8, 16}. Hence find the weight of the optimal tree. **arks)**